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High-speed algorithm for locating circular objects

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Abstract. *A high-speed method for circular object location is proposed. The Hough transform is a robust method for detecting parametrically described curves. However, when more than two parameters exist, evaluation of the possible parameters is very expensive. Through the use of the geometric property that states that the middle point of the hypotenuse of a right triangle is the center of the circumscribed circle, the proposed method can significantly reduce the time needed for evaluating all possible parameters. Experimental results are also given to show the effectiveness of the proposed method.*

1 Introduction

Circular object detection and location are very important in many application fields. In daily life, circular objects are very common, for example, coins, buttons, wheels, pills, etc. In image understanding and analysis, circular features usually play a key role. Thus, determining circular object location and estimation easily and efficiently is an important research topic.[1–3]

The Hough transform[3–5] is a robust way to detect circular features in a digital image. However, if circles of various radii exist in a digital image, a three-dimensional parameter space is required and a lot of time is needed to evaluate all possible candidates in the space. Generally speaking, use of the Hough transform to find circular objects causes the following problems:

1. A lot of storage is needed to store the count of each point in the parameter space.
2. A lot of computing time is needed to transform each feature point in the image space into many points in the parameter space.

Several modified versions of the Hough transform have been proposed.[4–7] These methods try to solve the previously mentioned problems, but they still cannot reduce much of the computing time required for candidate parameter evaluations. Therefore, developing a method for circular object location that is fast as well as easily applied to various practical applications is very important. In this paper, we present such a method. The method is based on the concept of the Hough transform and on the geometric property that states that the middle point of the hypotenuse of any right triangle is the center of the circumscribed circle of the triangle.

First, the proposed method extracts all object boundary points. Next, for each boundary point A, its two nearest boundary points B and C, with B in the horizontal direction and C in the vertical direction (see Fig. 1), are located. By means of the previously mentioned geometric property, a set of parameters (x_0,y_0,r_0) that may be the center and radius of a circular object is then obtained. Finally, based on the concept of the Hough transform, all circular objects are located. In the proposed method, for each boundary point, only one set of parameters (x_0,y_0,r_0) is evaluated, but in the known Hough-transform-based algorithms, many sets of parameters (x_0,y_0,r_0) are evaluated. Thus, the proposed method can greatly reduce the computing time.

The remainder of this paper is organized as follows. The proposed method is described in detail in Sec. 2. Next, in Sec. 3, the complexity of the proposed method is analyzed. Some experimental results are then presented in Sec. 4 to show the effectiveness of the proposed method. Finally, Sec. 5 presents some discussion and conclusions.

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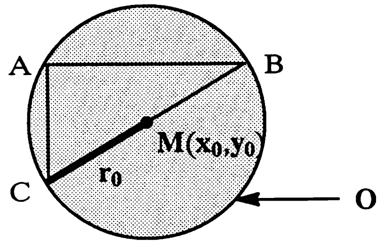


Fig. 1 The middle point M of the hypotenuse $\overline{BC}$ of the right triangle ΔABC is the center of the circumscribed circle O of ΔABC.

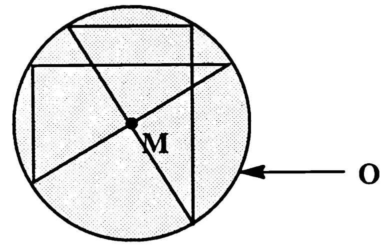


Fig. 2 The hypotenuse of any right triangle with its three vertices on a circle O must be a diameter and its middle point must be the center M of O.

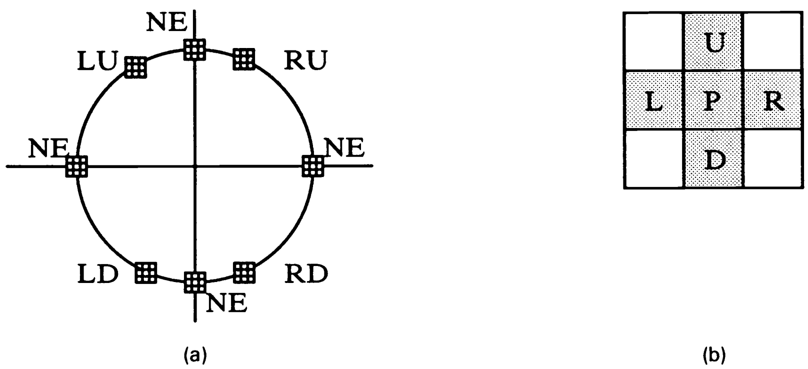


Fig. 3 An illustration for the classification of points on a circle: (a) five classes of the points on a circle and (b) the four neighbors of point P.

2 Proposed Method

The proposed method consists of two steps: boundary extraction and circular object location. In the boundary extraction step, all object boundary points in an image are extracted. In the circular object location step, which is based on the extracted boundary points and a geometric property, all circular objects are located. Before describing these two steps in detail, we first give some definitions, a theorem, and a criterion that are used in the proposed method.

2.1 *Definitions, Theorem, and Criterion*

Definition 1. A nearly binary image is an image in which each point has a gray level near either a constant g_1 or a constant g_2 ($g_1 < g_2$).

Definition 2. Let P be a black/white binary image; each black point in P is called an *object point* and each white point is called a *background point.*

Definition 3. In a black/white binary image, an object point whose eight connected neighbors include at least one background point is called a *boundary point.*

Theorem 1. Let ΔABC be a right triangle and $\overline{BC}$ be its hypotenuse. Let O be the circumscribed circle of ΔABC (see Fig. 1). Then the middle point M of $\overline{BC}$ is the center of circle O, and the length of $\overline{BM}$ is the radius of circle O.

Since theorem 1 is well known, its proof is omitted here. By means of theorem 1, we can obtain the following corollary immediately.

Corollary. Let O be a circle and M be the circle center. Then for any right triangle with its three vertices on circle O, as shown in Fig. 2, the hypotenuse must be a diameter of O, i.e., the middle point of the hypotenuse is M and half of the length of the hypotenuse is the radius of O.

Let O be a circle partitioned into five classes LU, RU, LD, RD, and NE [see Fig. 3(a)]. We can assign a point on O to belong to one class by inspecting its neighboring points. The detail is given by the following criterion.

Criterion 1. Let P be a point on a circle O and U, D, L, and R be its four connected neighbors [see Fig. 3(b)]. The class of P can be determined by the following rules:

Rule 1. If both L and U are not on O, P belongs to class LU.

Rule 2. If both R and U are not on O, P belongs to class RU.

Rule 3. If both L and D are not on O, P belongs to class LD.

Rule 4. If both R and D are not on O, P belongs to class RD.

Rule 5. If rules 1 through 4 are not true, P belongs to class NE.

Given the previous definitions, theorem, and criterion, we can describe the proposed method.

2.2 *Boundary Extraction Step*

In the boundary extraction step, the proposed method accepts

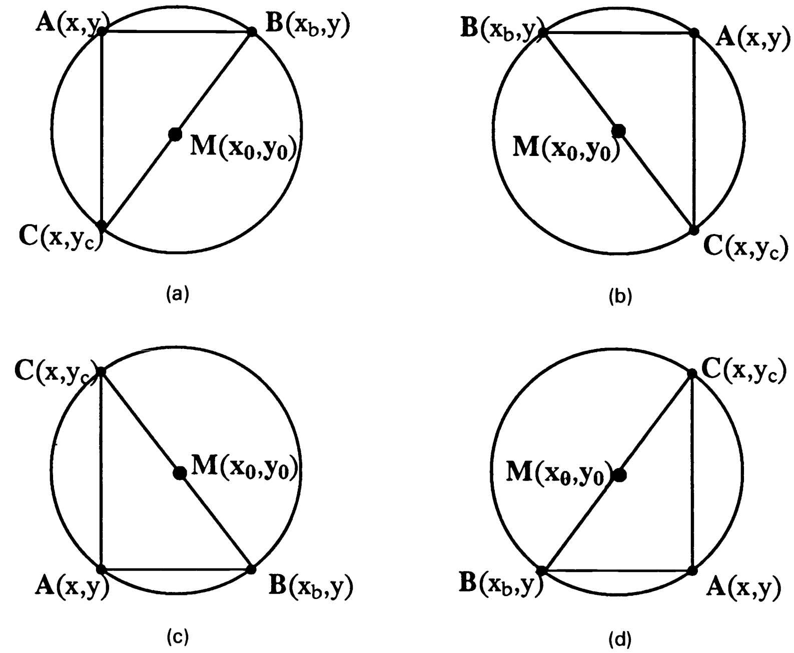


Fig. 4 The two nearest boundary points *B* and *C* of *A*: (a) *A* is in class *LU*, (b) *A* is in class *RU*, (c)*A* is in class *LD*, and (d) *A* is in class *RD*.

a nearly binary image as input data and converts it into a black/white binary image by means of the entropy-based binary thresholding algorithm.[8] The algorithm first takes a threshold value h that maximizes the following function:

$$\Psi_{(h)} = -\sum_{g\leq h} p_h^{(1)}(g)\ \log p_h^{(1)}(g) - \sum_{g>h} p_h^{(2)}(g)\ \log p_h^{(2)}(g)\ ,$$

where

$$p_h^{(1)}(g) = \frac{p(g)}{\sum_{g\leq h} p(g)}\ ,$$

$$p_h^{(2)}(g) = \frac{p(g)}{\sum_{g>h} p(g)}\ ,$$

and $p(g)$ is the frequency of gray level g in the input image f. Then a binary image f_b is constructed by

$$f_b(x,y) = \begin{cases} 1 & \text{if } f(x,y)\leq h \\ 0 & \text{otherwise} \end{cases}\ .$$

Based on the binary version, all the object boundary points can be extracted by checking each object point $P(x,y)$, i.e., $f_b(x,y)=1$. If at least one of the eight connected neighbors of P has a value of zero, P is considered to be a boundary point.

2.3 *Circular Object Location Step*

Based on the result from the boundary extraction step, the circular object location step locates all circles by means of the concept of the Hough transform and a geometry property. Initially, the proposed method establishes an array $AR(x,y,r)$ with $AR(x,y,r)=0$ for each entry (x,y,r). Then, for each boundary point $A(x,y)$, the class of A is first determined according to criterion 1. Next, its two nearest boundary points $B(x_b,y)$ (if A is in class LU or LD, B is in the right-horizontal direction, otherwise, it is in the left-horizontal direction) and $C(x,y_c)$ (if A is in class LU or RU, C is in the down-vertical direction, otherwise, it is in the up-vertical direction) are taken (see Fig. 4). The middle point $M(x_0,y_0)$ of $\overline{BC}$ and the length (r_0) of $\overline{BM}$ are obtained by the following two equations:

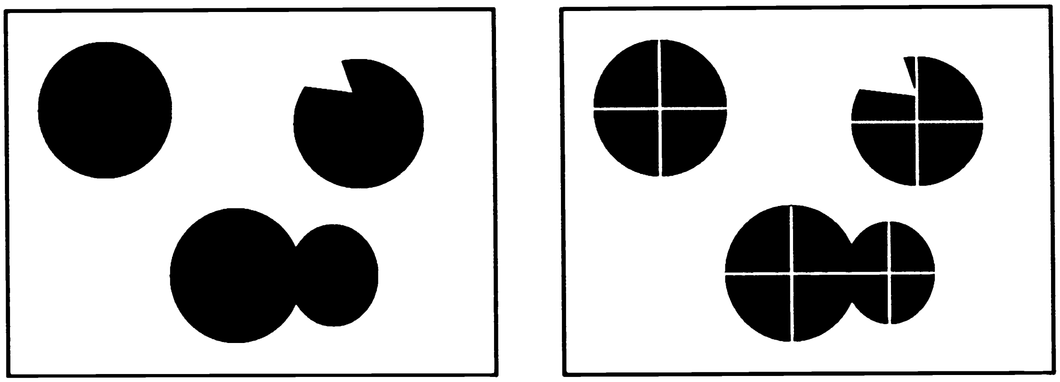
$$(x_0,y_0) = [(x+x_b)/2,(y+y_b)/2] \tag{1}$$

$$r_0 = [(x-x_0)^2 + (y-y_0)^2]^{1/2}\ . \tag{2}$$

Then, (x_0,y_0,r_0) is considered to be a set of possible parameters for a circle and the corresponding value $AR(x_0,y_0,r_0)$ is increased by one. Note that when A belongs to class NE, or B and C cannot be found, we do not process any further. After all boundary points are processed, a local peak-detection algorithm is finally applied to the array AR. Each of the peaks is considered as the parameters of one circle.

Here, we explain the correctness of the step. Assume that a circle O with center (x_0,y_0) and radius r_0 exists. Then for each point A on O, we can usually find its two nearest points B and C on O, according to the method of this step. By corollary 1, we know that the middle point M of $\overline{BC}$ is the center (x_0,y_0) of O and the length of $\overline{BM}$ is the radius (r_0) of O. Thus, the value $AR(x_0,y_0,r_0)$ increases by one. Since there are many points on O and each point contributes one to $AR(x_0,y_0,r_0)$, the value $AR(x_0,y_0,r_0)$ is large when the step ends. Hence, by finding the local peak, the circle is found.

Note that there are many local peak-finding algorithms.[3] Here, a simple method is used to locate local peaks. It searches the parameter space. If a point (x_0,y_0,r_0) with its associated value $AR(x_0,y_0,r_0)$ larger than a pregiven number and higher than those of its neighbors {each point $(x,y,r)\neq(x_0,y_0,r_0)$ with $x\in[x_0-1,\ x_0+1]$, $y\in[y_0-1,\ y_0+1]$, and $r\in[r_0-1,\ r_0+1]$



(a) (b)

Fig. 5 Result of applying the proposed method to a synthetic image: (a) a synthetic image and (b) the result of applying the proposed method to (a) with each white line segment standing for a diameter of the detected circular objects.

is called a neighbor of $(x_0,y_0,r_0)\}$ is found, it is thought to represent the set of parameters of a circular object with center (x_0,y_0) and radius r_0. (The value of the pregiven number can be determined according to r_0; in our experiments, $\lceil \pi r_0/4 \rceil$ is taken, which means that a circular object located by the proposed method must have at least one-eighth of its boundary points that could be used to produce its parameters.)

3 Complexity Analysis

In this section, we discuss the computational complexity of the proposed method and compare it with the standard Hough transform.[9] For convenience of discussion, some symbols are defined to represent the amount of computation:

t_a = the computation time of one addition or subtraction

t_m = the computation time of one multiplication or division

t_q = the computation time of one square root

t_s = the computation time of one bit shift.

For the standard Hough transform for circle detection, each circle $O(x_0,y_0,r_0)$ is represented by the following equation:

$$(x-x_0)^2+(y-y_0)^2=r_0^2 \ .$$

If both parameters x_0 and y_0 are quantized into q cells, then for each boundary point (x,y), we can obtain q^2 values of parameter r_0 corresponding to q^2 different pairs of (x_0,y_0), and each obtained parameter r_0 can be resolved by $[(x-x_0)^2+(y-y_0)^2]^{1/2}$. This means that for each boundary point, q^2 sets of parameters (x_0,y_0,r_0) need to be located, and the computation time is

$$q^2 \times (3t_a+2t_m+t_q) \ .$$

On the other hand, for each boundary point $A(x,y)$, the proposed method only needs to evaluate one set of parameters (x_0,y_0,r_0) by Eqs. (1) and (2). Here, the computation time is

$$5t_a+2t_m+2t_s+t_q \ .$$

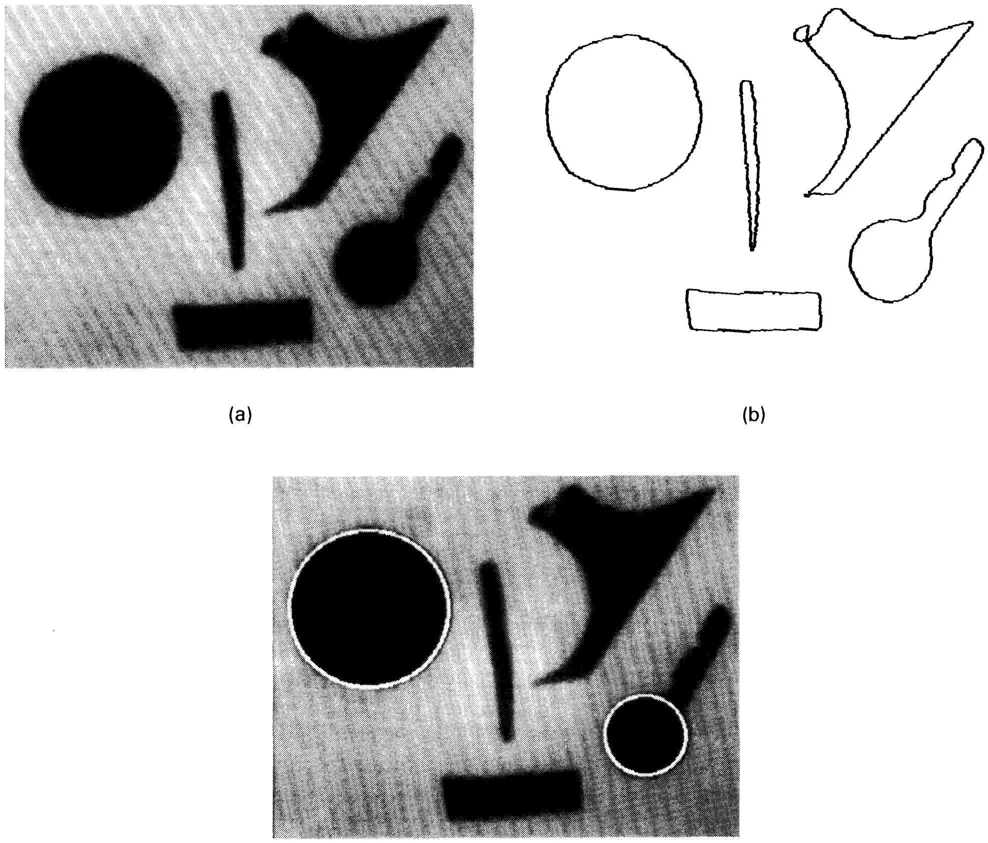
Since the computational complexity of the standard Hough transform depends on q, when q is a large number (e.g., 512), the computational complexity makes the Hough transform very time consuming to calculate. On the contrary, the computational complexity of the proposed method is independent of q. Since the time needed by multiplication and square-root operations dominates that needed by addition and shift, the speed of the proposed method is about q^2 times that of the standard Hough transform. For example, if there are 1627 boundary points in an input image, the proposed method implemented on a SUN workstation spends only 0.17 s in the accumulation step. But the standard Hough transform with $q=100$ will consume more than 1500 s.

4 Experimental Results

In this section, we present some experimental results. We tested several images containing perfect circular objects, defective circular objects, occluded circular objects, and other shapes. Figure 5(a) includes four synthetic objects: a perfect circular object, two partially occluded circular objects, and a broken circular object. Figure 5(b) is the result of applying the proposed method to Fig. 5(a); each white line segment stands for a diameter of the detected circular objects. From this figure, we can see that the proposed method works as anticipated. Figure 6(a) shows a real image including one separate coin, one key, and other shape objects. Figure 6(b) is the result of boundary extraction. In Fig. 6(c), the final result is superimposed on the original image with white points standing for the boundaries of the detected result. Two circular objects are located with radii of 58 and 30. The figure shows that the coin and the head part of the key were located successfully.

Figure 7(a) shows a noisy real image. Figure 7(b) is the result of applying the entropy-based binary thresholding algorithm. In order to prevent the noise in a circular object from intercepting the location of the two nearest boundary points that form a right triangle, a noise-cleaning filter is used. This filter contains two steps:

Step 1. In the black/white image, each background point is checked to see if it is enveloped by eight back-



(a) (b)

(c)

Fig. 6 Result of applying the proposed method to a real image: (a) a real image, (b) the result of the boundary extraction, and (c) the final result of the proposed method with two white circles standing for two extracted circular objects (coin and key head).

ground points. If the answer is no, it is replaced by an object point.

Step 2. After step 1, each object point is checked to see if it is enveloped by eight object points. If the answer is no, it is replaced by a background point.

The filter causes our proposed method to work normally when an input image contains noise. Figure 7(c) is the result of applying the filter to Fig. 7(b). The final result is shown in Fig. 7(d), which is the same as Fig. 6(c). All circular objects have been well extracted.

From these experimental results, we can see that partially occluded or defective circular objects can be detected and located by our method. But if a circular object has a bad defect, our method cannot extract it. This is because some points on the boundary of the circular object cannot form a right triangle.

5 Conclusions and Discussion

In this paper, we have proposed a high-speed method to locate circular objects. Based on a geometric property, for each boundary point A, we only need to evaluate one set of parameters representing the center and the radius of a circle that possibly passes A. Hence, the proposed method is much faster than the existing Hough-transform-based algorithms for detecting circular objects. In addition, the proposed approach is simple and can be easily implemented.

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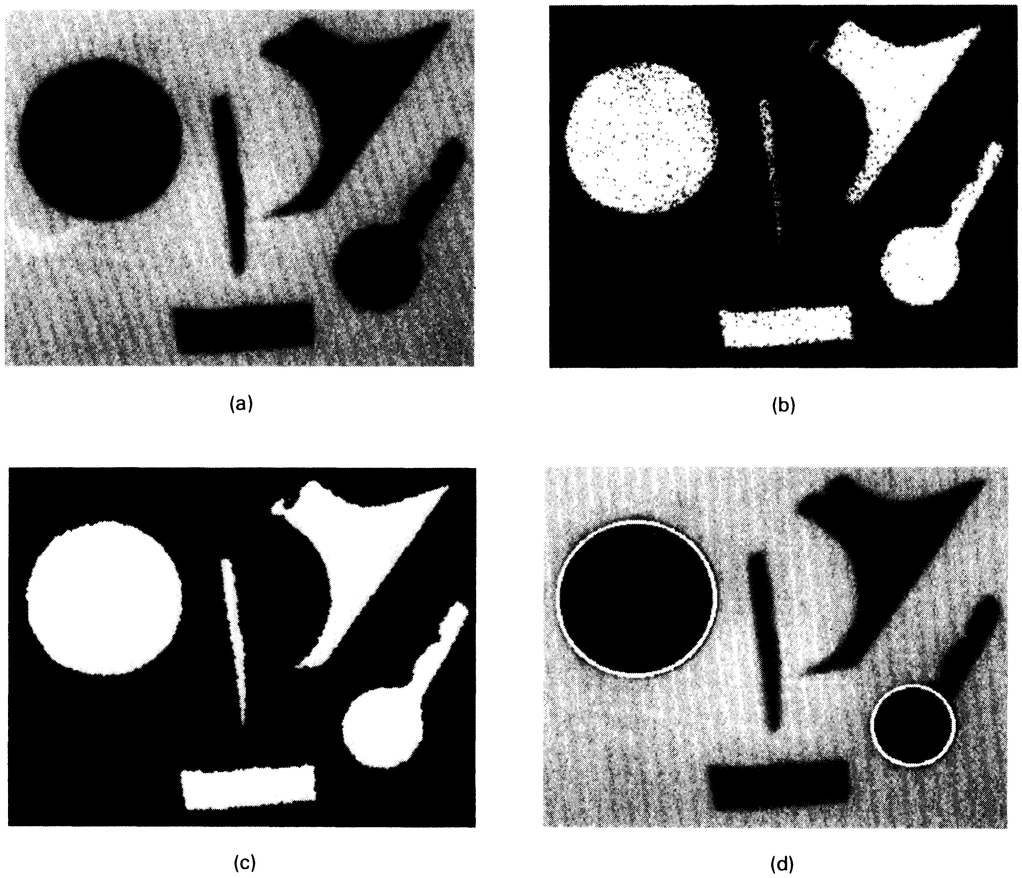

Fig. 7 Result of applying the proposed method to a noisy real image: (a) a noisy image, (b) the result of applying the entropy-based thresholding method to (a), (c) the result of applying the noise-cleaning filter to (b), and (d) the final result of the proposed method.

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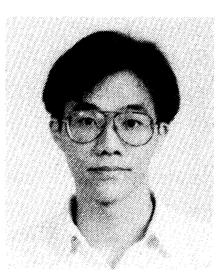

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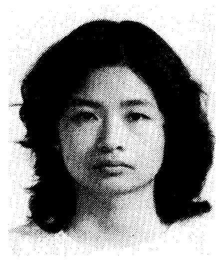

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